



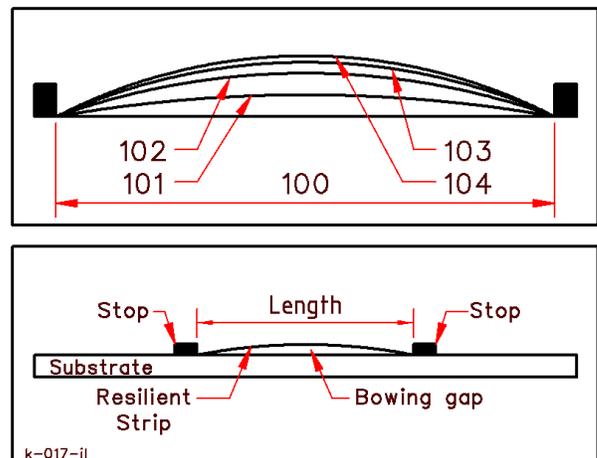
Mechanical Expansion Amplifier (US Patent 7,707,896)

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Magnetometer Underwater Detection Range, A Mechanical Expansion Amplifier Application

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10-Jul-18, minor revisions, see Errata for change list
 29-May-14, correction: In Table 1, the Gotland's magnetic signature suppression affects the magnetic moment directly. The detection distance is reduced as the cubed root rather than proportionally. The corrected detection range is greater.
 18-May-14, update
 04-Jan-10, update
 24-Apr-09, update

Reminder: The accuracy of the simple calculation herein (Ref C) is restricted to a line at right angles to the side of a cylindrical object pointing north. See Ref M for a complete vector solution.

1.0 An Ultra-Sensitive Underwater Magnetometer

Magnetic Anomaly Detection (MAD) employs magnetometers to detect very small changes in the earth's magnetic field. They are used for geophysical mineral and oil exploration, archeology, environmental surveys, ordnance and weapons detection (UXO), maritime intrusion detection, Anti-Submarine Warfare (ASW), and earth science experiments.

As List 1 shows, compact, low power, temperature tolerant magnetometers such as the flux-gate design lack sensitivity, while the sensitive instruments based on molten potassium or cryogenic superconducting quantum interference devices (SQUIDs) require bulky insulation and significant resources to maintain their operating temperature.

The Mechanical Expansion Amplifier (MEA) configured as a magnetometer provides a low-powered, ultra-sensitive magnetometer that can operate at any temperature in the -40C to +85C range with little change in sensitivity. The inherent noise limit is lower than that of the SQUIDs!

* MEA-based magnetometer:	ambient	.025e-15T/√Hz
* cryogenic SQUID:	4°K	3e-15T/√Hz
* electron resonance, molten potassium:	335°K	50e-15T/√Hz
* Overhauser proton precession:	300°K	10,000e-15T/√Hz
* proton precession:	300°K	300,000e-15T/√Hz
* flux-gate:	ambient	1,000,000e-15T/√Hz
* The earth's magnetic field:		58,000,000,000e-15T/√Hz

List 1: Sensitivity of competing magnetometer technologies

The wide operating temperature range means the MEA magnetometer can operate at ocean or lake temperature. The ultra-sensitive instrument is practical for harbor and shore picket lines, marine or aerial towed arrays, buoys, and semi-autonomous search vehicles.

This paper and Ref C, "Magnetic Field Perturbation from an Under-Sea Object", derive equations to estimate the range at which small underwater vehicles of various compositions can be passively detected.

The sensitivity for the Benchmark instrument is found from the simulation detailed in Ref A, "Benchmark Magnetometer Performance". It achieves a sensitivity of $25.0e-18$ Tesla in a 1.0 Hertz bandwidth. Since passing vehicles stay in range for several seconds, the 1.0 Hertz instrumentation bandwidth is adequate.

The sections in slide show Ref B, "Installing an Atto-Tesla Magnetometer", discuss noise suppression techniques and material selection criteria. The Benchmark design can be augmented with noise suppression for nearby electrical currents, temperature changes, accelerations, and temporal fluctuations in the earth's magnetic field.

2.0 Passive Magnetic Detection of Underwater Objects

The degree to which a material attracts or repels magnetic flux is called susceptibility, customarily represented by χ . A paramagnetic material attracts and concentrates magnetic flux inside itself. A diamagnetic material repels magnetic flux.

Common experience associates magnetic properties primarily with iron and nickel. However, almost all materials have magnetic susceptibilities, albeit a million times smaller, making their magnetism unnoticeable in everyday experience.

Water is slightly diamagnetic, reducing the earth's magnetic field strength by 9.1 parts per million. At a sufficient distance beneath the air interface, the magnetic field is locally uniform and typically changes slowly with location.

If a void (vacuum) is placed in the uniform underwater field, magnetic flux is attracted to it. Its susceptibility is zero and that of water is negative ($-9.034e-6$), making the void slightly paramagnetic with respect to its water environment.

The external magnetic flux is neither created nor destroyed by an object's susceptibility. The increased flux in the void is obtained by depleting the flux outside. The lines of constant flux density bend toward the void. The flux magnitude outside is diminished.

Either magnitude or direction (or both) can be used to detect the distortion of a local magnetic field by a newly introduced object.

The MEA-based magnetometers measure small changes (perturbations) in the earth's magnetic field rather than its absolute magnitude. Measuring deviations from the local normal simplifies the instrument and calculations.

For further simplification, note that the perturbed underwater magnetic field is indistinguishable from that caused by a similarly shaped object in a vacuum, if the object has a paramagnetic susceptibility equal in magnitude to that of water, $+9.034e-6$. It is conceptually convenient to reference the susceptibility of objects to that of the water environment rather than vacuum. This is done by subtracting the susceptibility of water from that of the object.

3.0 Calculation Method

Before a detection range can be calculated, some decisions about the test object and environment must be settled.

Test object: The objects of interest will vary widely in size and material content depending on the interests of the user. Instead of investing in the difficult magnetic field calculation for a specific vessel or object shape, choose a simple one. Then show how its size and net susceptibility scale the detection range.

For this very sensitive instrument, the size of the object should be small compared to the maximum detection distance, less than 10%. At greater distances, the shape of the object has only a minor influence on the magnetic field magnitude and direction.

Choose the smallest likely object, a miniature autonomous submarine or frogman-guided underwater sled. Approximate the object with a cylinder having a 1.0 square meter base area and 6 meter length (or an equivalent prism). For convenient analysis, the axis of the cylinder is aligned with the direction of the local geomagnetic field.

Objects of mixed composition have a net susceptibility, proportioned by volume, which adequately determines the detection range. The locations of the material components in the object can be ignored.

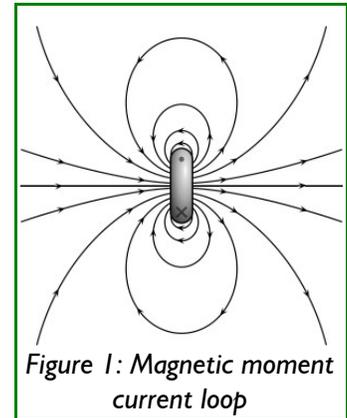
Environmental noise: As with size and shape, environmental noise and the degree of its suppression vary widely with application. For this range example, use the only noise known for sure, the inherent noise of the instrument.

Assume all environmental noises are suppressed below the inherent noise. If the application requires a maximum range beyond that calculated, it is unlikely to be attained with the current technology. If the range is more than adequate, some of the more expensive noise suppression enhancements may be scaled down.

Magnetic moment method: The magnetic moment concept provides a convenient way to estimate distant magnetic fields. A single wire loop contains a circulating current (Figure 1).

Although the field's magnitude and direction require complex calculations in general, simple and accurate formulas are known for two special directions. One calculates the field along the axis of the loop and the other calculates the field radially from the loop center. Both equations are in terms of a magnetic moment.

Figure 2 shows the special-case equations for the magnetic flux density generated by a current loop magnetic moment. The field drops off rapidly with distance to the third power. The detection range is about 25% greater (1.26) in the radial direction than in the axial.



The equations are valid near or far.

The magnetic moment can be found from the axis equation if the flux density is known at some point on the shared axis. If found, either equation can find the field at a distant point in its special direction.

Placement: For mathematical convenience, place the loop at the center of the cylinder with its axis aligned to that of the cylinder and the geomagnetic field. Set the arbitrary loop diameter equal to that of the cylinder.

Magnetic Object Detection Underwater	Known current loop fields
Along axis of current loop:	$axial_b = \frac{mm \mu_0}{2 distx^3}$
Radially from loop center:	$radial_b = \frac{mm \mu_0}{distx^3}$
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Figure 2: Magnetic moment flux densities (Tesla)

Although the current loop equations are accurate near or far, a single magnetic moment may not model its object accurately at short distances. But, as the distance increases, the true and calculated fields rapidly converge. The exact shape of the object or current loop makes little difference for distant magnetic fields.

Field point equation: As shown in Ref C, the simple loop geometry and symmetry allow a solution for the magnetic flux density on axis from basic magnetic principles. Figure 3 shows the equation obtained.

Magnetic flux density on axis (B):	$b_on_axis = \frac{i_loop \mu_0 obj_rad^2}{2 (obj_rad^2 + disty^2)^{3/2}}$
Magnetic moment definition:	$mm = loop_current * loop_area$

Figure 3: Equation for on-axis field

One known field point: The object's axial magnetic symmetry center is aligned with the earth's local magnetic field direction. When so aligned, the field direction does not deviate from its normal direction as it passes through the object. The on-axis flux magnitude just outside the object is that of the earth while that just inside is increased or decreased by the susceptibility change.

Figure 4 shows the Figure 3 equations combined with the flux change point to give the magnetic moment sought:

Magnetic Object Detection Underwater	Flux density at cylinder end
Flux density change at cylinder end (Tesla):	$b_earth (susc_obj - susc_h2o)$
Magnetic moment evaluated at obj_len/2:	$\frac{2 \pi b_earth \left(obj_rad^2 + \frac{obj_len^2}{4} \right)^{3/2} (susc_obj - susc_h2o)}{\mu_0}$
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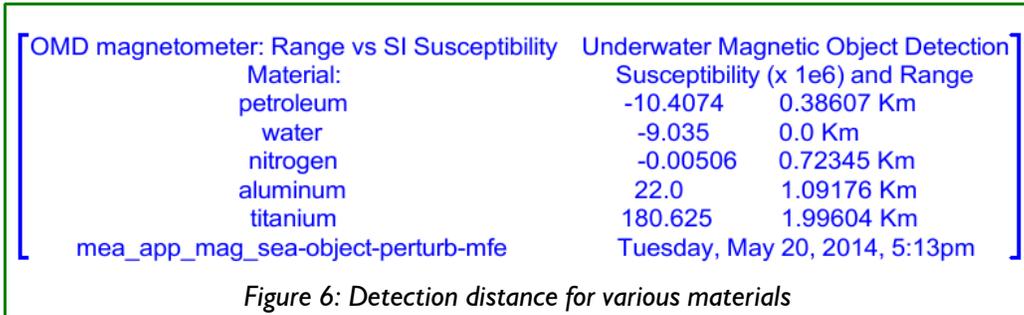
Figure 4: Magnetic moment for underwater cylinder

Magnetic flux density: All the terms on the right side of Figure 5 are known if the magnetometer lies on one of the special directions.

Magnetic moment giving net field change:	Magnetic Object Detection Underwater
Magnetic flux density on axis (Tesla):	$\frac{mm \mu_0}{2 disty^3}$
Perturbation flux density, axial (Tesla):	$\frac{\pi b_earth \left(obj_rad^2 + \frac{obj_len^2}{4} \right)^{3/2} (susc_obj - susc_h2o)}{disty^3}$
Perturbation flux density, radial (Tesla):	$\frac{2 \pi b_earth \left(obj_rad^2 + \frac{obj_len^2}{4} \right)^{3/2} (susc_obj - susc_h2o)}{distx^3}$
Where:	
Magnetic moment:	mm (Amp*m^2)
Earth's magnetic field:	b_earth (Tesla)
Magnetic susceptibility, water:	susc_h20 (#)
Magnetic susceptibility, object:	susc_obj (#)
Permeability of empty space:	mu0 (Henry/m)
Axial distance from object center:	distx, disty (m)
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Figure 5: Distant flux density in special directions

Detection range solution: Solving the Figure 5 equations for distance requires a flux density for the left side. The magnetometer's noise-equivalent-magnetic-flux gives the theoretical maximum detection distance, but a practical false alarm rate requires a field perhaps 10 times more. The plot which follows shows the detection distance for a test object of 6 cubic meters volume. Figure 6 gives the susceptibilities and exact distances:



The plot of Figure 7 shows the effect of susceptibility for objects of the same size. A 1.0m diameter 6m long tank might contain fuel (petroleum) or be empty (air). The metal objects at great distance have the same magnetic effect as a much larger object containing the same amount of metal. The titanium cylinder weighs 28230Kg. A small submarine weighs 50 times more, so the cylinder represents a midget submarine.

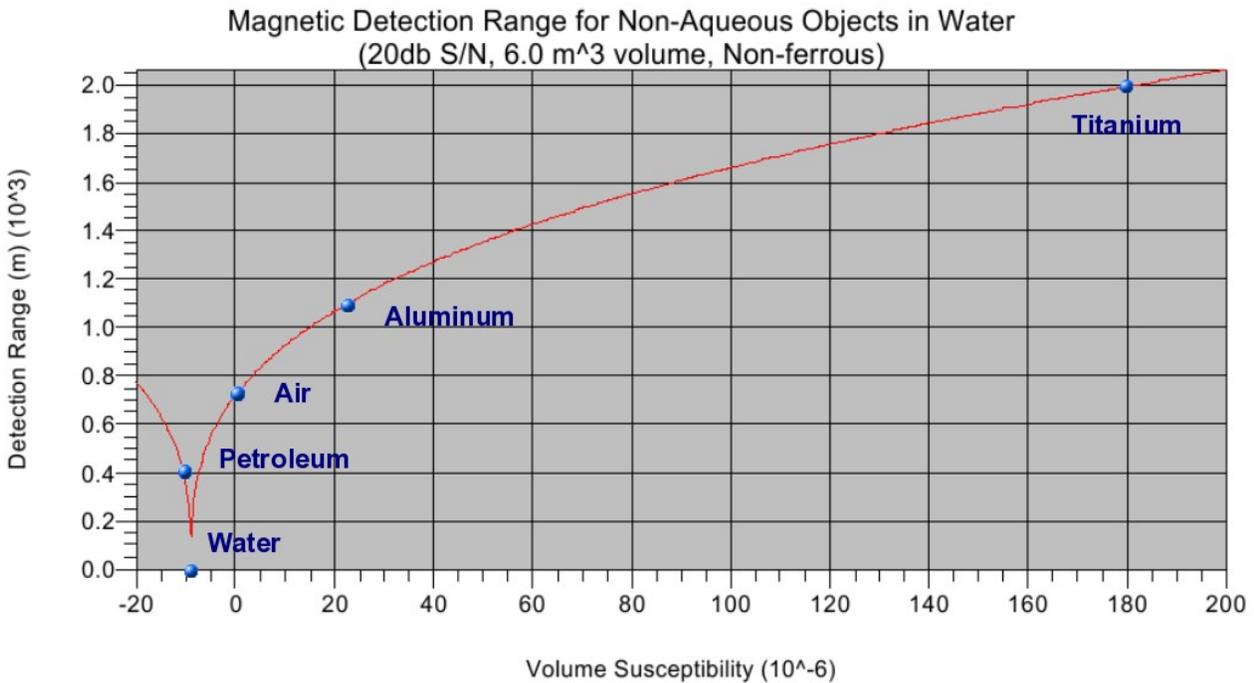


Figure 7: Underwater detection range

Discussion: The accuracy of Figure 7 depends on the detection distance being much larger than the object's dimensions. How well does that requirement hold?

For the petroleum, the largest object dimension is 6m and the detection distance is 386m for a ratio of 6/386= 1.6%, well below the 10% limit. Note that the detection distance gives a signal to noise ratio of 10-to-1 (20db) for a low false alarm rate.

4.0 Example: Gotland-Class Submarine

A desk calculator or spreadsheet is sufficient to evaluate the detection range equation for other shapes and materials. SI units require volume susceptibilities rather than molar or mass susceptibilities.

The environmental noise level is very likely to limit sensitivity before the ultimate MEA inherent noise level is reached. However, the range decreases slowly with increased noise. To calculate the noise suppression needed, rearrange the range equation to calculate the noise level allowed and evaluate it at the minimum acceptable range. The calculated noise divided by the inherent noise estimates how much the environmental noise must be suppressed.

Equations 1 includes both the symbolic equation and a partially evaluated version:

Magnetic Object Detection Underwater		Detection distance
Radial range:	$\text{distr} = \frac{\pi^{1/3} b_{\text{earth}}^{1/3} \sqrt{4 \text{obj_rad}^2 + \text{obj_len}^2} (\text{susc_obj} - \text{susc_h2o})^{1/3}}{4^{1/3} \text{noise_equiv_mag_flux}^{1/3}}$	
Axial range:	$\text{disty} = \frac{\pi^{1/3} b_{\text{earth}}^{1/3} \sqrt{4 \text{obj_rad}^2 + \text{obj_len}^2} (\text{susc_obj} - \text{susc_h2o})^{1/3}}{2 \text{noise_equiv_mag_flux}^{1/3}}$	
For:	SI Units	
Earth's local magnetic field:	b_earth = 58.0e-6 Tesla	
Susceptibility of water:	susc_h2o = -9.035e-6	
Suggested noise-equiv-mag-flux:	2.47214e-17 Tesla/Hz	
Signal-to-noise ratio (SNR):	10.0	
Object dimensions (cylinder):	end area= 1.0 m^2, length= 6.0 m	
Radial range:	$\text{distr} = \frac{0.03571 \sqrt{4.0 \text{obj_rad}^2 + \text{obj_len}^2} (\text{susc_obj} + 9.03501\text{e-6})^{1/3}}{\text{noise_equiv_mag_flux}^{1/3}}$	
Axial range:	$\text{disty} = \frac{0.02835 \sqrt{4.0 \text{obj_rad}^2 + \text{obj_len}^2} (\text{susc_obj} + 9.03501\text{e-6})^{1/3}}{\text{noise_equiv_mag_flux}^{1/3}}$	
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Equations 1: Detection range for other shapes and materials

As an example, consider a stealthy Gotland-class submarine. Assume the hull material has a magnetic susceptibility similar to that of Hastalloy stainless steel, $+2000e-6/m^3$. For simplicity, the remainder of the volume has the susceptibility of air.

The submarine is equipped with electromagnets to reduce its magnetic signature. Assume the compensation is 99% effective in masking its effect on the earth's magnetic field.



The MEA magnetometer is assumed to be equipped with a complete suite of noise suppression systems, allowing it to operate at its inherent noise level. (Ref B.)

To reduce false alarms, the detection range is based on a magnetic change equal to 10 times the inherent noise of the magnetometer.

Parameter	Value	Units	Formula	Note
Length (L)	60.4	m		
Diameter (D)	6.2	m		
Mass	1.49E+06	Kg		
Hastelloy X density (ρ)	8.22E+03	Kg/m ³	stainless steel	
Hastelloy X permeability (μ)	1.002		(maximum)	
Hastelloy X susceptibility (χ_{ss})	2.00E-03		$\chi_{ss} = \mu - 1.0$	1
Hastelloy X volume (V _{ss})	1.81E+02	m ³	$V_{ss} = \text{mass} / \rho$	2
Total volume (V _{tot})	1.82E+03	m ³	$V_{tot} = \pi * (D/2)^2 * L$	3
Air volume (V _{air})	1.64E+03	m ³	$V_{air} = V_{tot} - V_{ss}$	4
Air (N ₂) susceptibility (χ_{air})	5.00E-09			
Average susceptibility (χ_{ave})	1.99E-04		$\chi_{ave} = (\chi_{ss} * V_{ss} + \chi_{air} * V_{air}) / V_{tot}$	5
Noise_equiv_mag_flux	2.47E-17	T/ $\sqrt{\text{Hz}}$	Ref A	
Signal-to-noise ratio	10		20db	
Magnetic field suppression	99.00	%	HMS Gotland electromagnets	
Detection range	4.41	Km/ $\sqrt{\text{Hz}}$	distx = Right angle distance	6,7

Table 1: Maximum detection range, Gotland-class (from Wikipedia)

Calculation notes:

1. $= \chi_{ss} - 1.0$
2. $= V_{ss} / \rho$
3. $= 3.14159 * (D/2)^2 * L$
4. $= V_{tot} - V_{ss}$
5. $= (\chi_{ss} * V_{ss} + \chi_{air} * V_{air}) / V_{tot}$
6. $= 0.001 * 0.03571 * \sqrt{(\chi_{ave}^2 + \text{noise}^2) * ((1.0 - \text{supp}) / 100) * (\text{snr} + 9.03501e-6) / \text{snr}}$

$$(\langle B_{14} \rangle * \langle B_{13} \rangle)^{1/3}$$

7. The 0.001 converts to kilometers and 0.03571 is from Equations 1 above.

5.0 Magnetometer Deployment Suggestions

Vector analysis of a magnetometer's response to a passing object has revealed a very useful configuration. When two distant MEA-based magnetometers "look" at each other across a channel, establishing a "line-of-sight", they can resolve the exact moment a moving object crosses the line, where along the line it crossed, and the speed it was going. With two additional magnetometers, the class of the object can be determined in many cases.

The unique feature of the MEA magnetometer that provides the course information is its cosine squared angular response. Since the flux direction sensing is independent of the magnitude multiplier, the response has a zero-crossing and a slope. These combine with the known spatial separation of magnetometers and the temporal separation of response maxima to provide many analysis possibilities.

The instruments have about the size, weight, and power requirements of a flux-gate magnetometer with similar angular coverage.

Gradient measurement: Unlike magnetometers that respond to the total geomagnetic field, the MEA magnetometer does not need two magnetometer locations to measure a gradient. Its deviations from zero give the direction and magnitude of the gradient change.

Noise suppression: The two primary noise sources in many locations will be seismic or subsonic man-made noises and unbalanced changes in local electric currents. The slide show "Installing an Atto-Tesla Magnetometer", Ref B, makes some suggestions.

Simulation studies: The vector-based simulator "Response to Passing Magnetic Objects", Ref M, can be used with any magnetometer technology.

6.0 Appendix: Strains to Tesla Conversion

The MEA design for the magnetometer is simulated in Ref A, "Benchmark Magnetometer Performance". The strain sensitivity is $1.80e-19$ (m/m)/ $\sqrt{\text{Hz}}$. How is that related to an equivalent magnetic flux density?

- The Benchmark design noise-equivalent strain: $1.80e-19$ (m/m)/ $\sqrt{\text{Hz}}$
- Terfenol-D strain coefficient: $9.1e-9$ (m/m)/(A/m) Ref J
- What magnetic flux density (Tesla) creates a magnetic field strength equal to 1.0 Ampere-turn/meter?

- * In the traditional representation, H is magnetic field strength in Amp-Turns/meter and B is the magnetic flux density in Weber/m² or Tesla
- * $B/H = \mu_0$ where $\mu_0 \approx 4\pi \times 1.0e-7$ Henry/m in air
- * $B = \mu_0 \cdot H = (4\pi \times 1.0e-7 \text{ Hy/m}) \cdot (1.0 \text{ A/m})$
- * $H_y = \text{Weber/Ampere-turn}$
- * $B = 4\pi \times 1.0e-7 ((\text{Wb/A})/\text{m}) \cdot (\text{A/m}) = 4\pi \times 1.0e-7 \text{ Wb/m}^2 = 4\pi \times 1.0e-7 \text{ Tesla}$
- * $9.1e-9 \text{ (m/m)} / (\text{A/m}) \cdot (\text{A/m}) / (4\pi \times 1.0e-7 \text{ T}) \cdot B_{\text{min}} = 1.8e-19 \text{ (m/m)} / \sqrt{\text{Hz}}$
- * $B_{\text{min}} = 2.48e-17 \text{ Tesla} / \sqrt{\text{Hz}}$

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